STAT 134: Concepts of Probability

Classical Poker Probability Problems

University of California at Berkeley

Last Edit Date: 01/10/2023

Disclaimer and Term of Use:

1. We do not guarantee the accuracy and completeness of the summary content. Some of the course material may not be included, and some of the content in the summary may not be correct. You should use this file properly and legally. We are not responsible for any results from using this file.
2. This work is licensed under a Creative Commons Attribution 4.0 International License.

Poker Problem

There are 52 cards in a poker.
4 suits: diamonds ♦ ×13 spades ♠ ×13
clubs ♣ ×13 hearts ♥ ×13
kinds: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A (13 in total)

1. Straight Flush.

Contains 5 cards of sequential rank, all of the same suit.

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10
7	8	9	10	J
8	9	10	J	Q
9	10	J	Q	K
10	J	Q	K	A
A	2	3	4	5

can be: clubs, diamonds, spades, hearts

There are 10 possible combanitions, each combanition has 4 possible suits.

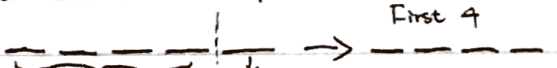
As a result, there can be 10×4 possible straight flush.

$\binom{10}{1} \cdot \binom{4}{1} = 10 \times 4 = 40$

$$\mathbb{P}(\text{Straight flush}) = \frac{\binom{10}{1} \cdot \binom{4}{1}}{\binom{52}{5}}$$

2. Four of a Kind

Contains 4 cards of one rank and one card of another.



52 − 4 = 48 possible outcomes

A	A	A	A	
2	2	2	2	
3	3	3	3	
4	4	4	4	
5	5	5	5	
6	6	6	6	
⋮	⋮	⋮	⋮	
Q	Q	Q	Q	
K	K	K	K	

13

For each possible combanition, all 4 suits are contained.

There are 13 possible outcomes for the first 4 cards,

and 48 possible outcomes for the other card.

As a result, there can be 13×48 possible four of a kind.

$$\mathbb{P}(\text{Four of a kind}) = \frac{\binom{13}{1} \cdot \binom{48}{1}}{\binom{52}{5}}$$

3. Full House

Contains three card of one rank and two cards of another rank.

A	A	A	
2	2	2	
3	3	3	
4	4	4	
⋮	⋮	⋮	
Q	Q	Q	
K	K	K	

13

Each combanition, the first one can have 4 possible suits,
second one can have 3 possible suits,
third one can have 2 possible suits.

So it can be A♦ A♣ A♠, A♦ A♣ A♥, A♦ A♠ A♥, A♣ A♠ A♥ — 4 for the first combanition.

As a result, there are 13×4 possible outcomes for the first 3 cards.

Can be any two cards from the rest card, but cannot have the same rank as first 3.

If A A A, then _ _.

2	2
3	3
4	4
⋮	⋮
Q	Q
K	K

12

Each combination, the first one can have 4 possible suits.

Second one can have 3 possible suits.

So it can be 2♣ 2♠, 2♣ 2♦, 2♣ 2♥, 2♠ 2♦, 2♠ 2♥, 2♦ 2♥ — 6 for the first combanition.

As a result, there are 12×6 possible outcomes for last 2 cards.

$$\mathbb{P}(\text{Full House}) = \frac{\binom{13}{1} \cdot \binom{4}{3} \times \binom{12}{1} \cdot \binom{4}{2}}{\binom{52}{5}}$$

4. Flush.

Contains 5 cards all the same suit, not all of sequential rank.

_ _ _ _ _ There are 4 suits: spades ♠ ×13, diamonds ♦ ×13, clubs ♣ ×13, hearts ♥ ×13

For each suit, we choose 5 cards. So there are $\binom{13}{5}$ outcomes for each suit.

As a result, there are $4 \times \binom{13}{5}$ possible outcomes in total.

$$\boxed{\mathbb{P}(\text{Flush}) = \frac{\binom{4}{1}\cdot\binom{13}{5}}{\binom{52}{5}}}$$

* this contains outcomes that all 5 cards are consequtive.

For flush does not contain all 5 cards are consequtive. We can have:

In each suit, we have 10 combanitions that have 5 consequitive rankes.

$$10\left\{\begin{array}{ccccc} A & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 9 & 10 & J & Q & K \end{array}\right.$$

As a result, there are $\binom{13}{5} - 10$ outcomes to avoid concequtive ranks in each suit.

$$\boxed{\mathbb{P}(\text{Flush}) = \frac{\binom{4}{1}\cdot\left[\binom{13}{5}-\binom{10}{1}\right]}{\binom{52}{5}}}$$

5. Straight

Contains 5 cards that are ranked concequtively, not all same suit.

_ _ _ _ _ Consider ranks, there can have 10 combanitions.

$$10\left\{\begin{array}{ccccc} A & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 9 & 10 & J & Q & K \end{array}\right.$$

Each rank can have 4 possible suits. So there are 4^5 possible combanitions for each concequtive rank combanitions. There are $4^5 \cdot 10$ total outcomes.

$$\boxed{\mathbb{P}(\text{Straight}) = \frac{\binom{10}{1}\cdot 4^5}{\binom{52}{5}}}$$

* this contains 5 cards that have same suits

For straight does not contain 5 cards with same suits. We can have:

Since not allowed 4 of 5 cards have same suit, so we have to exclude the cases that there are 4 cards in the same suit. There are 4 possible outcomes for all 5 same suit in each rank combanition.

So for each rank combanition, for the first 4 cards, we can only have $4\times4\times4\times4$ ways to assign suits.

For the last number, there are 4 possible suits can take. But we need to subtract the all the same suit, so there are (4^5-4) possible suits combanitions in each rank combanition.

$$\boxed{\mathbb{P}(\text{straight}) = \frac{\binom{10}{1}\cdot(4^5-4)}{\binom{52}{5}}}$$

6. Three of a kind

Contains 3 same rank. Other two can be anything but not the same rank.

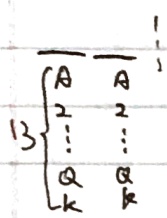
_ _ _ ; If A A A, then _ _ $\binom{12}{2}$.

$$13\left\{\begin{array}{ccc} A & A & A \\ 2 & 2 & 2 \\ \vdots & \vdots & \vdots \\ Q & Q & Q \\ K & K & K \end{array}\right. \qquad \binom{12}{2}\left\{\begin{array}{cc} 2 & 3 \\ 2 & 4 \\ \vdots & \vdots \\ K & K \\ \vdots & \vdots \end{array}\right.$$

$$\boxed{\mathbb{P}(\text{Three of a kind}) = \frac{\binom{4}{3}\cdot\binom{13}{1}\times\binom{12}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}}$$

7. Two Pairs

Two same rank, another two same rank.



Permutation:
- First two: $\binom{13}{1}\cdot\binom{4}{2}$
- Second two: $\binom{12}{1}\cdot\binom{4}{2}\cdot\frac{1}{2!}$
- Last one: $\binom{11}{1}\cdot\binom{4}{1}$

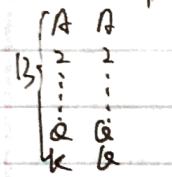
OR

Combination:
- First Four: $\binom{13}{2}\binom{4}{2}\binom{4}{2}$
- Last one: $\binom{11}{1}\binom{4}{1}$

$$\mathbb{P}(\text{Two Pairs}) = \frac{\binom{13}{1}\cdot\binom{4}{2}\times\binom{12}{1}\cdot\binom{4}{2}\times\binom{11}{1}\binom{4}{1}}{\binom{52}{5}\cdot 2!} = \frac{\binom{13}{2}\cdot\binom{4}{2}\binom{4}{2}\times\binom{11}{1}\cdot\binom{4}{1}}{\binom{52}{2}}$$

8. One Pair

Contains two same rank.



Combination:
- First two: $\binom{13}{1}\binom{4}{2}$
- Last three: $\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$

$$\mathbb{P}(\text{One Pair}) = \frac{\binom{13}{1}\binom{4}{2}\times\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$$

For Permutation

$$\mathbb{P}(\text{One Pair}) = \frac{\binom{13}{1}\binom{4}{2}\times\binom{12}{1}\binom{4}{1}\times\binom{11}{1}\binom{4}{1}\times\binom{9}{1}(\text{illegible})}{\binom{52}{5}\cdot 3!}$$

9. High Card

Cannot have combinations above.

Cannot have straight, so minus $\binom{10}{1}$ for each combination.

Cannot have flush, so minus (4) for each combination.

$[\binom{13}{5}-\binom{10}{1}]\cdot(4^5-4)$ possible outcomes.

$$\mathbb{P}(\text{High Card}) = \frac{[\binom{13}{5}-\binom{10}{1}]\cdot(4^5-4)}{\binom{52}{5}}$$